which the heat diffuses remains the same for equal spatial increments. The solution of Kuo *et al.* [1] yields too low temperatures during the initial stage, whereas it overshoots the exact solution when it is extended to larger time intervals. Both effects are probably due to a not appropriately chosen approximation formula. The considerable difference between the numerical solution and the solution obtained under the assumption of a spatial uniform temperature is to be expected for the used value of *Bi*.

In Fig. 2 results are represented for an Al_2O_3-particle with $r_s = 5$ mm, obtained from the approximate solutions equations (15) and (16), and the numerical solution. The corresponding time interval is $t = 20$ s, which exceeds the validity range of equation (15) by a factor of 25. The parameter varied is T_g. Again, the solution of Kuo and Summerfield [2] achieves excellent agreement with the exact solution as long as it does not exceed its validity range too far. However, the deviation from the numerical solution increases with increasing T_g and increasing *Fo*, whereas the solutions obtained from equation (16) yield good agreement with the numerical solution for $Fo \geqslant Fo_\delta$.

Figure 3 shows the temperature profiles inside a solid propellant-particle with $r_s = 5$ mm. The corresponding time interval is 1.7 s. The good agreement with the numerical solution during the initial stage slowly decreases with increasing penetration of the thermal wave. Although equation (6) and equation (12) provide identical results for $Fo = Fo_\delta$, there occurs a rapid displacement of the temperature profile in the environment of the center shortly after the beginning of the second stage. This deviation decreases with increasing time while in the outer layers beneath the particle surface there is always good agreement with the numerical solution.

Figure 4 again shows results for an Al_2O_3-particle with $r_s = 5$ mm. The corresponding time interval is $t = 10$ s. In this case, T_g and *Bi* are exponential functions of *Fo*. For $0 \leqslant Fo \leqslant Fo_\delta$ the approximate solutions are obtained from equation (7) while for $Fo \geqslant Fo_\delta$ equation (13) is solved. These approximate solutions, the numerical solution, T_g, and *Bi* are presented. It can be gathered that there is again good agreement between the numerical solution and the approximate solution assembled from equations (7) and (13).

6. CONCLUDING REMARKS

The approximation formula given by Kuo and Summerfield [2] turned out to yield the best agreement to the numerical solution, however, its validity range is limited. For that reason an extended solution, linked to that formula, was derived. For the case that both T_g and *Bi* are constants, the corresponding ODE's could be solved analytically. The surface temperature obtained by these two approximation formulas agrees very well with the numerical solution, whereas in the center of a pellet deviations can occur.

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ON THE EFFECT OF A LARGE TEMPERATURE DIFFERENCE ON THE VELOCITY AND TEMPERATURE PROFILES FOR THE TURBULENT FLOW OF AIR IN A TUBE

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NOMENCLATURE

- c_p specific heat of the gas at constant pressure [J kg^{-1} K^{-1}]
- c_{px} specific heat of the gas at constant pressure evaluated at T_x [J kg^{-1} K^{-1}]
- c_{pw} specific heat of the gas at constant pressure evaluated at the wall temperature T_w [J kg^{-1} K^{-1}]
- q_w heat flux to or from the gas at the wall [W m^{-2}]
- T gas temperature [K]
- T_B gas bulk temperature [K]
- T_w wall temperature [K]
- T_x geometrical mean between wall and gas bulk temperature [K]
- t_w^+ dimensionless gas temperature with gas properties evaluated at T_w
- t_x^+ dimensionless gas temperature with the gas properties evaluated at T_x
- u_w^* friction velocity with the gas density evaluated at T_w [m s^{-1}]
- u_x^* friction velocity with the gas density evaluated at T_x [m s^{-1}]
- u_w^+ dimensionless gas velocity with the gas density evaluated at T_w
- u_x^+ dimensionless gas velocity with the gas density evaluated at T_x
- y radial distance from the wall of the considered point [m]

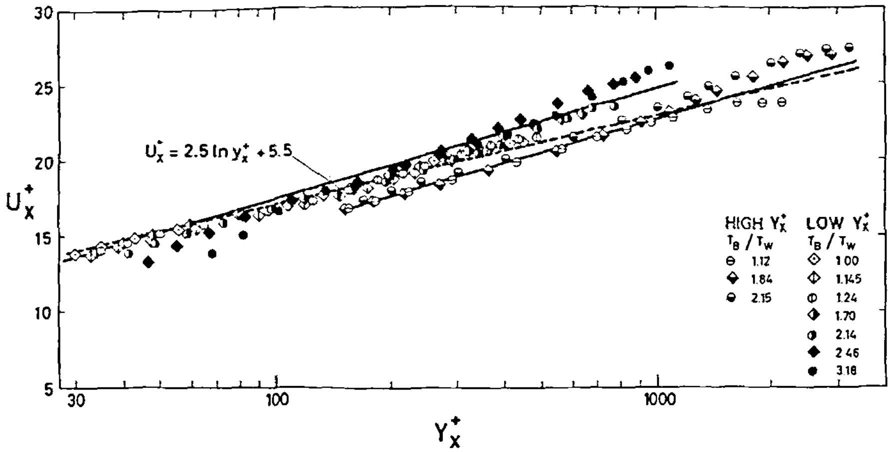

FIG. 1. Dimensionless gas velocity vs dimensionless radial distance from the wall (data of Mizushina *et al.*, correlation method of Dalle Donne and Meerwald).

y_w^+ dimensionless radial distance from the wall with gas properties evaluated at the temperature T_w

y_x^+ dimensionless radial distance from the wall with the gas density evaluated at T_x and the gas kinematic viscosity evaluated at T_w

Greek symbols

ν_w kinematic viscosity of the gas evaluated at the wall temperature T_w [$m^2\ s^{-1}$]

ρ gas density [$kg\ m^{-3}$]

ρ_x gas density evaluated at the temperature T_x [$kg\ m^{-3}$]

ρ_w gas density evaluated at the temperature T_w [$kg\ m^{-3}$]

τ_w shear stress at the wall [$kg\ m^{-1}\ s^{-2}$]

1. INTRODUCTION

THE PROBLEM of a gas flowing inside a duct in the presence of a large temperature difference between the wall and the gas has been experimentally investigated by many authors before. A large amount of information on friction and heat transfer coefficients is available [1–8]. However, not much is known on the gas temperature distribution normal to the wall or on the gas velocity distribution in the presence of large temperature differences between the wall and the fluid. The few measurements performed so far are for low temperature differences [9, 10]. In this case the fluid properties may be considered constant in a cross section normal to the flow direction and the effect of the variable gas physical properties cannot be investigated. In a more recent paper, Mizushina *et al.* [11] show the results of velocity and temperature measurements in air in turbulent flow inside a tube in the presence of large temperature differences between a hot gas and the wall. The data are correlated in terms of 'universal' velocity and temperature profiles, i.e. u_w^+ and t_w^+ as functions of y_w^+, whereby the gas physical properties appearing in these parameters have been evaluated at the wall temperature T_w. The measurements of Mizushina *et al.* show that with this type of correlation, the T_w/T_B effect, i.e. the effect of the variability of the physical properties in the cross section normal to the flow direction, is still very large, especially at high values of y_w^+, i.e. the profiles differ quite considerably for different values of T_w/T_B. In ref. [7], Dalle Donne and Meerwald suggested another way of correlating the gas velocity distribution in turbulent flow in the presence of a large temperature difference. So far this method is not based directly on velocity measurements, but rather it was suggested by their measurements of friction coefficients for air flow in an annulus and by the friction factor correlation of Taylor [6] for flow of gases in tubes at high temperatures. In this note a comparison is performed of the correlation method of Dalle Donne and Meerwald directly with the velocity (and temperature) distribution measurements of Mizushina *et al.* [11].

2. ANALYSIS

The definitions used in ref. [11] are

$$y_w^+ = \frac{y u_w^*}{\nu_w}, \tag{1}$$

$$u_w^+ = \frac{u}{u_w^*}, \tag{2}$$

$$t_w^+ = \frac{(T-T_w)c_{pw}u_w^*\rho_w}{q_w}, \tag{3}$$

$$u_w^* = (\tau_w/\rho_w)^{1/2}. \tag{4a}$$

In ref. [11], T is always higher than T_w, thus t_w^+ is positive if the heat flux, q_w, at the wall is considered positive with heat flow from the gas to the wall. In the experiment of ref. [7], the heat is flowing from the wall to the gas. However, the definition of t_w^+ of equation (3) may be maintained because q_w is in this case negative and t_w^+ remains still positive ($T < T_w$). The friction factor measurements of refs. [6, 7] suggest that the friction velocity should be evaluated with the following equation to eliminate the T_w/T_B effect:

$$u_x^* = (\tau_w/\rho_x)^{1/2} \tag{4b}$$

with

$$\rho_x = \rho_x[T_x = (T_w T_B)^{1/2}]. \tag{5}$$

(See equation (11) of ref. [7]), while the kinematic viscosity of the gas, ν_w, is evaluated at the temperature of the wall T_w,

$$y_x^+ = \frac{y u_x^*}{\nu_w}, \tag{6}$$

$$u_x^+ = \frac{u}{u_x^*}. \tag{7}$$

An analogous definition for the temperature is introduced here,

$$t_x^+ = \frac{(T-T_w)c_{px}u_x^*\rho_x}{q_w} \tag{8}$$

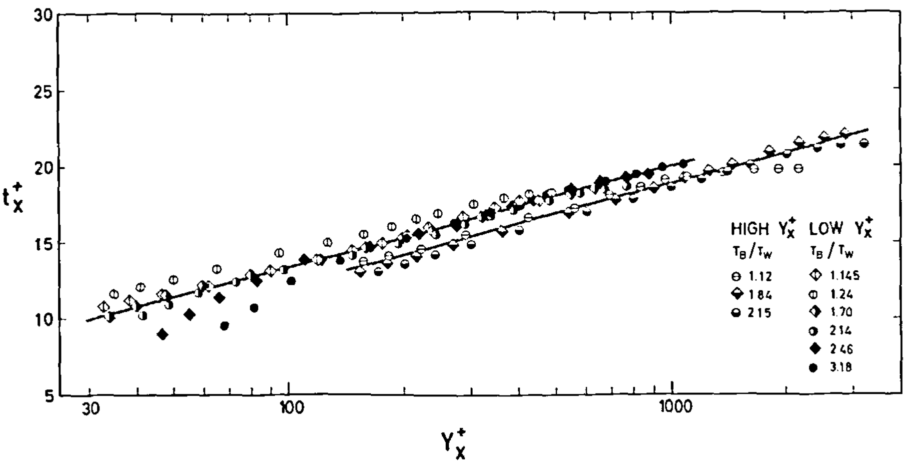

FIG. 2. Dimensionless gas temperature vs dimensionless radial distance from the wall (data of Mizushina *et al.*, correlation method suggested in the present paper).

where the specific heat of the gas, c_{px}, is evaluated at the gas bulk temperature, T_x. Indeed the variations of c_p with temperature for air and other gases are relatively low in the practical ranges of temperature and in any case the specific heat, like the density, is related to the heat transport along the tube as well as to the heat transfer from the wall, thus the evaluation temperature T_x seems to be appropriate. For air in the range of temperature 300–1300 K one has [2]

$$\rho \propto T^{-1}, \quad (9)$$

$$c_p \propto T^{0.119}. \quad (10)$$

Thus for air one obtains the following relationships:

$$u_x^* = u_w^* \left(\frac{T_B}{T_w}\right)^{0.25}, \quad (11)$$

$$u_x^+ = u_w^+/(T_B/T_w)^{0.25}, \quad (12)$$

$$y_x^+ = y_w^+ \left(\frac{T_B}{T_w}\right)^{0.25}, \quad (13)$$

$$t_x^+ = t_w^+ \bigg/ \left(\frac{T_B}{T_w}\right)^{0.19}. \quad (14)$$

Figures 1 and 2 show the experimental data of Mizushina *et al.* recalculated in the form u_x^+ vs y_x^+ and t_x^+ vs y_x^+, respectively. (The original numerical data of the measurements of Mizushina *et al.* have been obtained from the authors in tabulated form.) From the two diagrams one can observe that:

(1) In both cases the experimental points form two groups: one for high values and the other for low values of y_x^+.

(2) At the center of the tube (for the maximum values of y_x^+ of each run) the values of u_x^+ tend to be higher than the values predicted by the universal velocity profile, shown in Fig. 1. However this is a fairly well known fact, also for isothermal flow.

(3) The most important fact is, for our concern, that the T_w/T_B effect has been eliminated, i.e. no systematic differences between points at different T_w/T_B values can be seen anymore. At very low values of y_w^+ (less than 100), a small systematic T_w/T_B effect is still detectable, probably due to the fact that the buffer layer, between the fully turbulent and laminar layers, begins at this point.

One can thus conclude that the measurements of Mizushina *et al.* [11] confirm the correlation method of Dalle Donne and Meerwald [7], and that an analogous correlation method holds for the temperature profiles as well.

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